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A note on the holographic interpretation of string theory backgrounds with varying flux

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Abstract: We discuss the field theory interpretation (via holographic duality) of some recently-discovered string theory solutions with varying flux, focusing on four-dimensional theories with $\mathcal{N} = 2$ supersymmetry and with $\mathcal{N} = 1$ supersymmetry which arise as the near-horizon limits of "fractional D3-branes". We argue that in the $\mathcal{N} = 2$ case the best interpretation of the varying flux in field theory is via a Higgs mechanism reducing the rank of the gauge group, and that there is no need to invoke a duality to explain the varying flux in this case. We discuss why a similar interpretation does not seem to apply to the $\mathcal{N} = 1$ case of Klebanov and Strassler, which was interpreted as a "duality cascade". However, we suggest that it might apply to different vacua of the same theory, such as the one constructed by Pando Zayas and Tseytlin.

Keywords: AdS/CFT Correspondance, Supersymmetry and Duality.

Contents

1. Introduction and summary

It is very interesting to generalize the AdS/CFT correspondence [1, 2, 3] (see [4] for a review) to theories without conformal invariance. On the field theory side non-conformal theories can have richer dynamics, and are more directly related to experiment. On the string theory side, such a generalization enables a holographic interpretation of additional backgrounds beyond anti-de Sitter spaces.

The simplest way to obtain such a generalization is to start from a conformal field theory with a known AdS dual, and then deform it by relevant operators, and/or go to its moduli space (if it has one), thus breaking conformal invariance explicitly or spontaneously. In these cases the string theory background is still asymptotically AdS, with known behaviors near the boundary for various fields, corresponding to the deformations and vacuum expectation values (VEVs) we turn on in the field theory. Various examples of this type have been studied, one of the more interesting ones (which is related in various limits to $\mathcal{N} = 1$ SYM and to pure Yang-Mills theory) being the mass deformation of the $\mathcal{N} = 4$ SYM theory, whose string theory dual was found in [5].

An alternative way to get non-conformal theories is to study gravitational backgrounds with a different asymptotic behavior. For example, backgrounds which asymptote to linear dilaton backgrounds were argued in [6] to be dual to "little string theories". In fact, this seems to be the only general (Lorentz-invariant) example which is well understood. Different asymptotic behaviors would correspond to theories whose UV behavior is neither that of a local field theory nor that of a "little string theory", and we do not know of any other possible behaviors for Lorentz-invariant theories.

An interesting example of a background with a different asymptotic behavior was found in [7] (following [8, 9]) by examining the near-horizon limit of N D3-branes and M fractional D3-branes at a conifold point. The background of [7] is completely non singular, but its asymptotic behavior is different from all previously known examples. In particular, the 5-form flux on the compact 5-cycle grows with the radial direction, and diverges at infinity, suggesting that the background may correspond to the large-N limit of some field theory (unlike the usual examples which are dual to field theories of finite N, though N may be large). The 2-point functions in this background [10] and its finite temperature behavior [11] seem to agree with this interpretation, and it would be interesting to make this interpretation more precise. Masses of some of the scalar particles in the theory corresponding to this background were recently computed in [12].

Several other examples with the same type of behavior have since been discovered. In [13, 14], various backgrounds in which the flux varies were discussed. Some of these backgrounds are asymptotically AdS, so they have a simple interpretation as a deformation of a conformal theory (and/or a point on its moduli space), but in the other backgrounds the flux diverges at infinity so they are of the same type as the background of [7] (though generically the divergence of the flux in these backgrounds is much worse than the logarithmic divergence of [7], so it is not clear if they are really of the same class). In [15, 16] the near-horizon limit of fractional branes at an $\mathbb{R}^4/\mathbb{Z}_2$ orbifold point was studied, and found to lead to a similar behavior of the flux.[1] The authors of [18] constructed another solution that has the same asymptotic behavior as the solution of [7], but a different structure in the interior of space, which contains a repulson-like singularity. If this singularity can be resolved this background should correspond to another vacuum of the same field theory as in [7]. Clearly, many other solutions with the same behavior can also be constructed by similar methods.

It seems natural to associate the changing flux with a decrease in the number of degrees of freedom of the field theory as one decreases the energy. The authors of [7] suggested that in the case with $\mathcal{N} = 1$ supersymmetry which they discussed,[2] this decrease comes from a series of duality transformations reducing the size of the gauge group. In this paper we would like to suggest that in other cases, such as the $\mathcal{N} = 2$ case of [15, 16], there is actually a different, simpler interpretation of this decrease. The theories involved in the backgrounds discussed here all have large moduli spaces, corresponding to putting the branes at other positions rather than the origin (before taking the near-horizon limit). We suggest that in the case of [15, 16], and perhaps also in some of the other cases, the gauge theory dual to the known string theory backgrounds is at a point on its moduli space such that the distribution of the branes exactly mirrors the source for the corresponding field in the string theory; namely,

[1] The same configuration was studied with different boundary conditions in [17].
[2] The background of [7] was shown to be supersymmetric in [14, 19].

that in the field theory there "really are" D3-branes where there is a source for the 5-form flux in the string theory (with obvious generalizations to theories in other dimensions). At this position in the moduli space the gauge group is spontaneously broken by the Higgs mechanism, leading naturally to a reduction in the size of the group. Of course, the gauge theory could also be at a point with zero VEVs (at least classically), but we claim that this point does not correspond to the known string theory duals. This seems to be the most naive interpretation of the varying flux, and we will see that in the $\mathcal{N} = 2$ theories of [15, 16] it seems to be implied by the VEVs we compute in such backgrounds.

In the case with $\mathcal{N} = 2$ supersymmetry discussed in [15, 16] we will be able to test this conjecture in various ways, since in this case we know how to define the theory as a limit of well-defined field theories, and since we can use the information from the effective action of wrapped D5-branes in the corresponding background. This case will be described in detail in section 2. We do not perform any new non-trivial computations, but just review how the existing results are consistent with the "Higgsing interpretation".

In section 3 we will discuss the case of [7] with $\mathcal{N} = 1$ supersymmetry. In this case it seems that the same interpretation does not apply, so the decreasing flux is probably best thought of as coming from a "duality cascade" as described in [7]. A different vacuum of the same theory was constructed in [18], and if the singularity there can be resolved, we suggest that in this vacuum the decrease in the size of the gauge group could perhaps be interpreted as Higgsing, like in the $\mathcal{N} = 2$ case. For the cases with $\mathcal{N} = 1$ supersymmetry the evidence for the identification of the position in the moduli space and for the interpretation of the varying flux is much weaker, and it would be nice to understand them better.

Similar methods can be used to analyze other cases, such as the backgrounds constructed in [13, 14], to see whether they are well described by a Higgsing interpretation, or if dualities are needed to explain the decrease in the flux. Of course, in the cases of $d \neq 4$ which are not dual to gauge theories the "Higgsing interpretation" does not literally involve a Higgs mechanism, but the relevant physics (in terms of where the theory sits in the moduli space) is completely analogous. We will not discuss these other cases here.

2. Theories related to branes on an $\mathbb{R}^4/\mathbb{Z}_2$ singularity

2.1 The conformal case

We will begin by discussing the theory of N D3-branes on an $\mathbb{R}^4/\mathbb{Z}_2$ singularity in type-IIB string theory. The low-energy theory on these branes is the $\mathrm{U}(N) \times \mathrm{U}(N)$ $\mathcal{N} = 2$ gauge theory with 2 bifundamental hypermultiplets. The sum of the inverse gauge couplings $\tau_i = 4\pi i/(g_{YM}^2)_i + \theta_i/2\pi$, $i = 1, 2$, is related to the string coupling

$\tau = i/g_s + \chi/2\pi$, while their difference is related to the integrals of the 2-form fields over the 2-cycle which vanishes at the orbifold singularity. The appearance of two gauge groups may be interpreted as arising from fractional D3-branes, or D5-branes (and anti-D5-branes) wrapped on the vanishing 2-cycle, as described in detail (for instance) in [15]. The beta functions of both $\mathrm{SU}(N)$ factors vanish, while the diagonal $\mathrm{U}(1)$ is free and decoupled, and the off-diagonal $\mathrm{U}(1)$ is free (and becomes a global symmetry) in the IR.

The classical moduli space of this field theory agrees precisely with the configurations of (possibly fractional) branes moving on the orbifold. There is a "Higgs branch" where the hypermultiplets obtain VEVs, in which case a combination of the adjoint fields becomes massive (while the other combination can also acquire a VEV). The moduli space of this branch is of the form $(\mathbb{R}^4/\mathbb{Z}_2 \times \mathbb{R}^2)^N/S_N$, corresponding to the motion of D3-branes in the background. On this branch there are no quantum corrections to the metric on the moduli space, both on the field theory side and on the string theory side. There is also a "Coulomb branch" in which the hypermultiplet VEVs vanish but the two adjoint fields (in the $\mathcal{N} = 2$ vector multiplets), φ_1 and φ_2, acquire VEVs, whose eigenvalues may be interpreted as positions of fractional D3-branes (or wrapped D5-branes). This branch is of the form $(\mathbb{R}^2)^N/S_N \times (\mathbb{R}^2)^N/S_N$. There are also mixed branches which correspond to configurations with both D3-branes and wrapped D5-branes in an obvious way.

The near-horizon limit of this brane configuration is related by the AdS/CFT correspondence [1, 2, 3, 20] to the non-trivial part of the low-energy field theory on the branes, which is the $\mathrm{SU}(N) \times \mathrm{SU}(N)$ $\mathcal{N} = 2$ supersymmetric gauge theory with 2 bifundamental hypermultiplets. The geometry of this near-horizon limit is [20] $AdS_5 \times S^5/\mathbb{Z}_2$, where the $\mathbb{Z}_2$ action leaves fixed an S^1 inside the S^5. The global symmetry of the field theory is $\mathrm{SU}(2)\times\mathrm{SU}(2)\times\mathrm{U}(1)_R\times\mathrm{U}(1)_B$. The first three factors are realized as $\mathrm{SO}(4) \times \mathrm{SO}(2)$ isometries of $S^5/\mathbb{Z}_2$, while the fourth corresponds to a gauge field from the twisted sector, as described below.[3]

The field theory has a global $\mathbb{Z}_2$ symmetry exchanging the two gauge groups (which is broken when they have different gauge couplings), and this may be identified with the $\mathbb{Z}_2$ symmetry of the orbifold, under which twisted sector states are charged. Thus, operators in the field theory which are symmetric under this $\mathbb{Z}_2$ are identified with untwisted sector states which propagate on the full $AdS_5 \times S^5/\mathbb{Z}_2$, while anti-symmetric operators are identified with twisted sector states which propagate only on the fixed line $AdS_5 \times S^1$. Generically the light states living on the fixed line are just a six-dimensional tensor multiplet. However, when the two 2-form fields integrated over the vanishing 2-cycle vanish, there are additional light states coming from D3-branes wrapped on the vanishing 2-cycle, which give rise to tensionless strings; in

[3] This gauge field can be thought of as the 4-form of type-IIB supergravity integrated over the 2-cycle which vanishes at the orbifold and over the fixed S^1.

the limit $g_s \to 0$ these strings give the A_1 $\mathcal{N} = (2,0)$ "little string theory" (see [21] for a review with references on "little string theories"). In the low-energy theory on AdS_5 these strings give rise to an SU(2) gauge symmetry; presumably this can be identified with an SU(2) global symmetry of the corresponding field theory which arises when one of the couplings becomes infinite.[4]

There is no general method to construct the moduli space of a field theory from its holographic dual. One has to construct separate solutions for each possible value of the VEVs (which is a different superselection sector in the field theory) and verify that they have the same UV behavior (so they correspond to the same field theory), with appropriate boundary values for the string theory modes that correspond to the fields which acquire expectation values. In the case of the $\mathrm{SU}(N) \times \mathrm{SU}(N)$ $\mathcal{N} = 2$ theory described above, it is not hard to construct solutions corresponding to any configuration in the moduli space.

First, let us examine configurations on the Higgs branch, corresponding to moving around the D3-branes. The description of these configurations is the same as that of the moduli space of the $\mathcal{N} = 4$ SYM theory. For the $\mathcal{N} = 4$ theory, the dual background is

$$ds^2 = Z^{-1/2}\eta_{\mu\nu}dx^\mu dx^\nu + Z^{1/2}dx^m dx^m \,, \tag{2.1}$$

where $\mu, \nu = 0, 1, 2, 3$, $m, n = 4, 5, 6, 7, 8, 9$, and $Z = 4\pi g_s N\alpha'^2/r^4$ with $r^2 = x^m x^m$. There is also a 5-form field proportional to the derivative of Z, which we will not write explicitly. To describe configurations on the Coulomb branch of this theory, one simply replaces Z by a more general harmonic function

$$Z = 4\pi g_s \alpha'^2 \sum_{j=1}^{N} \frac{1}{|\vec{x} - \vec{x}_j|^4} \,, \tag{2.2}$$

where the $\vec{x}_j$ are related to the field theory VEVs (the eigenvalues $\vec{\Phi}^j$ of the six matrices $\vec{\Phi}$) by $\vec{\Phi}^j = \vec{x}_j/2\pi\alpha'$. This is because the eigenvalue VEVs behave like D3-branes, which are sources for Z (appearing in the metric and in the 5-form). This identification can be tested by computing the VEVs of operators such as $\mathrm{tr}(\vec{\Phi}^n)$ in this background (see, for example, [22]). For general configurations on the Coulomb branch, this supergravity background is singular (corresponding to the fact that the field theory is free in the IR). Configurations where all branes are in big clumps have a good description in supergravity. Alternatively, configurations with large N but with only a small number of non-zero VEVs may be described in terms of D3-brane probes propagating in the background generated by the other branes, since their back-reaction is negligible in the large-N limit. For example, the configuration where only one VEV $\vec{\Phi}^1$ is non zero is dual to the string theory background with $Z = 4\pi g_s(N-1)\alpha'^2/r^4$, with a single D3-brane at the position $\vec{x} = 2\pi\alpha'\vec{\Phi}^1$.

[4] A U(1) subgroup of this SU(2) is the $\mathrm{U}(1)_B$ global symmetry which arises from the off-diagonal U(1) in $\mathrm{U}(N) \times \mathrm{U}(N)$.

The moduli space of the Higgs branch of the orbifold theory may be simply derived by orbifolding these backgrounds by $x^m \to -x^m, m = 6,7,8,9$. The harmonic functions are the same, except that we have to add also an image for every D3-brane [22], so that

$$Z = 4\pi g_s \alpha'^2 \sum_{j=1}^{N} \left(\frac{1}{|\vec{x} - \vec{x}_j|^4} + \frac{1}{|\vec{x} - \vec{\tilde{x}}_j|^4} \right), \tag{2.3}$$

where the reflection takes $x \to \tilde{x}$, and the total flux (now defined by integrating the 5-form over $S^5/\mathbb{Z}_2$) is still N.

To describe the Coulomb branch, corresponding to wrapped D5-branes, we have to take into account the fact that these are sources for the 3-form field strengths of type-IIB string theory, which are usually written in the combination $G_3 = F_3 - \tau H_3$. The wrapped D5-branes have to sit at the fixed point of the orbifold, so they only move in x_4 and x_5, and it will be convenient to label their position by the complex variable $z = x_4 + ix_5$. It turns out [15, 16] that for the moduli space configurations it is enough to turn on the components of the 2-form along the vanishing 2-cycle whose volume form we denote by ω_2, so that $G_3 = d(\theta\omega_2)$. θ may be written as $\theta_C - \tau\theta_B$, where $\theta_{B,C}$ are the integrals of the corresponding 2-forms on the vanishing 2-cycles, normalized to have periodicity 2π. One then finds [15, 16] a simple Laplace equation for θ, with positive-sign sources at the positions z_i of wrapped D5-branes and negative-sign sources at the positions $\tilde{z}_i$ of wrapped anti-D5-branes, and the solution is

$$\theta = 2i \left[\sum_{j=1}^{N} \ln(z - \tilde{z}_j) - \sum_{j=1}^{N} \ln(z - z_j) \right] - \theta_B^0 \tau \, , \tag{2.4}$$

where θ_B^0 denotes the value of θ_B in the theory at the origin of moduli space (which in the orbifold theory is $\theta_B^0 = \pi$ [23]).[5] The equation for Z is now more complicated than the Laplace equation, since the varying 2-form fields result in an additional source for the metric and for the 5-form field, $dF_5 = -F_3 \wedge H_3$ (recall that $H_3 = dB_2^{NS-NS}$, $F_3 = dC_2$ and $F_5 = dC_4 - C_2 \wedge H_3$, where C_k is the k-form RR field). This is in addition to the source coming from the direct coupling of the wrapped D5-branes to the 5-form field. The equation and its solution for coinciding wrapped D5-branes may be found in [16]. We will not need its explicit form here.

The 5-form flux varies in the solution because of the contribution of the 3-form fields to this flux, in addition to the naive variation coming from the D3-brane charge carried by the wrapped 5-branes. To get a well-defined solution one has to be careful that this flux does not become negative (of course, for supergravity to be valid we actually require that this flux, as well as the flux multiplied by the

[5] We could also have a non-zero θ_C^0, but this will not play an important role so we set $\theta_C^0 = 0$.

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string coupling, are much greater than one). Generally this puts some limit on the possible positions of the wrapped D5-branes. In the field theory the z_j correspond to the eigenvalues of the VEV of the first SU(N)-adjoint field φ_1, and the $\tilde{z}_j$ are the eigenvalues of the VEV of the second SU(N)-adjoint field φ_2. The moduli space metric of the field theory is now corrected (if $z_j \neq \tilde{z}_j$), both at 1-loop and from instanton effects, and the classical identification of the z_j with eigenvalues of the adjoint fields is no longer exact. The Seiberg-Witten curve [24, 25] for this field theory was found in [26], but (as far as we know) there is no explicit form for this curve in terms of the gauge-invariant operators in the field theory (which are classically related to the z_j).[6] Generally, the quantum corrections in the SW curve limit the possible values of the z_j. For example, in the pure SU(2) theory, the configuration with a vanishing (generalized) VEV a is not on the quantum moduli space. We expect that in the SU(N)$\times$SU(N) theory the quantum corrections arising from the SW curve will result in a positive 5-form flux for all the configurations on the quantum moduli space, at least whenever the supergravity approximation is valid. It would be very interesting to verify this prediction of the AdS/CFT correspondence directly.

2.2 The non-conformal case and its suggested interpretation

Instead of looking at just N D3-branes at the orbifold point, we could look at N D3-branes plus M wrapped D5-branes (if we have also wrapped anti-D5-branes we can replace a wrapped D5-brane and a wrapped anti-D5-brane by a D3-brane, so this is the most general case). The same arguments imply that the low-energy field theory in this case is the SU(N) $\times$ SU($N+M$) $\mathcal{N}=2$ gauge theory with 2 bifundamental hypermultiplets. However, for any $M > 0$ this field theory is not asymptotically free (the beta function of one of the gauge factors is positive and of the other is negative), so there is no limit where it can be decoupled from string theory and result in some well-defined dual background.

Nevertheless, people have boldly attempted to find duals for this theory, first perturbatively in M/N in [8] and later exactly (in the supergravity approximation) in [15, 16]. The solutions they found have 3-form fields of the form described above, with $\theta = -2iM\ln(z/z_0)$ for some (arbitrary) cutoff z_0. In these solutions there is an IR singularity at $|z| = r_e$, similar to the famous enhançon [29, 30, 31, 32], which is presumably resolved in a similar way. There is also a problem in the UV (large $|z|$), which is that the 5-form flux grows logarithmically there, and is not bounded

[6] The curve was found using the brane construction of these field theories. The other theories we discuss here also have brane constructions, but it is not clear to us how to use them to study the issues we discuss here. The one-loop and one-instanton corrections to the classical curve have been expressed in terms of gauge-invariant observables in the case of a single bifundamental hypermultiplet in [27], and presumably this can be generalized also to the conformal case of two bifundamental hypermultiplets, perhaps along the lines of [28], but this has not yet been done as far as we know.

by N, so it is not really possible to interpret these theories as dual to a particular $\mathrm{SU}(N) \times \mathrm{SU}(N+M)$ gauge theory. This problem is related to the fact that this gauge theory is not asymptotically free so it is not well defined.

Using the results of section 2.1, we can identify how these solutions arise from solutions of string theory whose field theory interpretation is known. If we look in the moduli space of the $\mathrm{SU}(N+M) \times \mathrm{SU}(N+M)$ theory at a configuration where M of the $\tilde{z}_j$ are equal to $(-z_0)$ (and all the other z_j's and $\tilde{z}_j$'s vanish), we have

$$\theta = -2iM \ln\left(\frac{z}{z+z_0}\right) - \theta_B^0 \tau\,. \tag{2.5}$$

In the classical field theory, in such a configuration, at energies below the scale $|z_0|/2\pi\alpha'$ we have the $\mathrm{SU}(N) \times \mathrm{SU}(N+M)$ gauge theory with two bifundamental hypermultiplets.[7] In the limit of $z_0 \to \infty$, keeping z constant, we get simply

$$\theta = -2iM \ln\left(\frac{z}{z_0}\right) - \theta_B^0 \tau\,, \tag{2.6}$$

as above. However, to be able to take $z_0 \to \infty$ we have to take $N \to \infty$ at the same time, if we want to avoid the 5-form flux becoming negative (and the solution becoming singular). Thus, the solutions may be identified with this large-N limit, and they do not make sense for finite N. As long as we do not take $z_0 \to \infty$ and we remain with $\theta = -2iM \ln(z/(z+z_0)) - \theta_B^0\tau$, we can have (as described in [15]) consistent solutions also for finite N, where the ratio between the radial coordinate r_e where the 5-form flux vanishes and $|z_0|$ behaves as $e^{-N\pi/g_s M}$ in the large-N limit.

For simplicity, let us focus now on the solution $\theta = -2iM \ln(z/z_0) - \theta_B^0\tau$ (remembering that it should be interpreted via a large-N limit as above). The supergravity equation for the 5-form field has a source proportional to the wedge product of the two 3-forms in the theory. This results in a D3-brane charge density in the solution, localized completely at the orbifold singularity, and proportional to $|\partial_z\theta|^2 = 4M^2/|z|^2$. The total 5-form flux localized between some circle of radius r_1 in the z-plane and a circle of radius $r_2 > r_1$ in this plane is given by $M \ln(g_s M r_2/\pi r_1)$. When we reduce the radius by a factor of $e^{\pi/g_s M}$, the flux is reduced by M, and $\theta_B \to \theta_B + 2\pi$. Since θ_B is periodic with period 2π, the solution is thus self-similar under such shifts, with an appropriate decrease of N and appropriate shifts resulting from the shift in θ.

What is the interpretation of this decrease of N? At first sight this seems to imply some sort of strange duality between the $\mathrm{SU}(N) \times \mathrm{SU}(N+M)$ gauge theory and the $\mathrm{SU}(N-M) \times \mathrm{SU}(N)$ theory that we get after $N \to N-M$, as suggested in a similar configuration in [7]. However, we would like to suggest that the interpretation in this case is much more mundane. We claim that the D3-brane flux in the string theory can be interpreted as corresponding to appropriate VEVs in the dual field

[7] There is also a decoupled $\mathrm{SU}(M)$ $\mathcal{N}=2$ SYM theory coming from the wrapped D5-branes at $z=-z_0$.

theory. Namely, we suggest that in the field theory the source for the 5-form can be thought of as coming not from 3-form fields (which have no analog in the field theory), but rather from an actual distribution of D3-branes and/or wrapped D5-branes (corresponding to the field theory being at a specific point in its moduli space). This means that in the field theory there should be M eigenvalues of each of the two adjoint fields which correspond to positions in the band between r_1 and $e^{\pi/g_s M}r_1$, for any r_1.[8] Note that we suggest that the eigenvalues of the two adjoint fields in this band are equal, $z_j = \tilde{z}_j$, so they are not a source for the 2-form fields θ. The full string theory backgrounds of [15, 16] may then be identified with the large-N limit of a configuration on the moduli space of the field theory which has M anti-D5-branes (= eigenvalues of φ_2) at some $(-z_0)$ (which goes to infinity in the large-N limit), M eigenvalues of φ_1 and φ_2 in each band between r and $e^{\pi/g_s M}r$, and finally (when we "run out of eigenvalues") M D5-branes at some radius $r_e \simeq |z_0|e^{-N\pi/g_s M^2}$ which constitute an enhançon. Depending on the exact location of the eigenvalues, in each band the gauge group $\mathrm{SU}(N+M)\times\mathrm{SU}(N)$ is spontaneously broken to some subgroup of $\mathrm{SU}(N) \times \mathrm{SU}(N-M) \times \mathrm{U}(M) \times \mathrm{U}(M)$. The fields of the extra $\mathrm{U}(M) \times \mathrm{U}(M)$ presumably correspond to some modes of the string theory (living both off and on the orbifold) which tend not to propagate to radii smaller than this band; we will discuss this further below. The moduli of the extra $\mathrm{U}(M) \times \mathrm{U}(M)$ are not directly visible in the string theory just like the original moduli were not visible, but we can make them visible by changing the solution ("moving around the wrapped 5-branes") as described above.

2.3 Pros and cons of the Higgsing interpretation

What is the evidence for this interpretation? First, we claim that it is the simplest possible interpretation of the background, which does not require any unknown dualities, but just simply translates the sources for the 5-form and 3-form fields to positions in the moduli space.

Another test of the validity of this identification comes from looking at the moduli space metric. The metric for a D3-brane probe, corresponding to moving eigenvalues of both adjoint matrices together, is flat and boring in the configuration we are discussing (since the dilaton and axion are constant). However, the metric for a wrapped D5-brane is more interesting, and was computed in [16]. Interestingly, this effective metric does not depend on the function Z, which appears in the background metric and 5-form but cancels out completely in the wrapped D5-brane metric. The result is simply

$$g_{ab} = \delta_{ab}\frac{1}{8\pi g_s}\theta_B = \delta_{ab}\frac{1}{8\pi g_s}\left(\theta_B^0 + 2Mg_s \ln\left|\frac{z}{z_0}\right|\right). \tag{2.7}$$

[8] In the supergravity limit we cannot say exactly where in the band these eigenvalues lie because the width of the band is very small, so we will leave this question open.

This is exactly the same result we get in the field theory, identifying the D5-brane position with an eigenvalue of φ_1, if we identify (as discussed in [15]) $4\pi/g^2_{\mathrm{SU}(N+M)} = \theta^0_B/2\pi g_s$. The logarithm in the field theory arises at 1-loop in the configuration described above, independently of where we put the D3-branes (= equal adjoint eigenvalues) which do not affect the 1-loop result. A non-renormalization theorem guarantees that there are no additional perturbative corrections to the moduli space metric. Additional non-perturbative corrections can arise from instantons, but are negligible (in the large-N, -M limit) as long as the effective coupling does not diverge, namely, whenever (2.7) is non zero.

However, the metric (2.7) does in fact vanish quite close to z_0, when

$$|z| = z_1 \equiv |z_0| e^{-\theta^0_B/2Mg_s}\,, \tag{2.8}$$

and it seems to become negative when $|z| < z_1$. How should we interpret this? From the supergravity point of view, this divergence of the effective coupling is very similar to the one which arises at an enhançon. Attempting to move a wrapped D5-brane beyond this radius leads to a non-supersymmetric configuration and thus costs energy. We suggest that, as in the enhançon case, the field theory configuration space does not allow the corresponding distribution of eigenvalues in which a single eigenvalue is taken below z_1. Since we do not have an explicit expression for the Seiberg-Witten curve in this case we cannot check this directly, but it seems to be a prediction of the AdS/CFT correspondence. We will call the circle $|z| = z_1$ a generalized enhançon ring, and we will discuss the physics there in more detail below. Note that this generalized enhançon ring is different from the original enhançon in that it is not constituted of branes but of supergravity fields (and some additional fields described in section 2.4), and the solution does not change significantly as one goes through it. In particular, as we discuss in a moment, other probes can go through this generalized enhançon without feeling it.

Even though we cannot take a wrapped D5-brane to $|z| < z_1$, the gravitational background there is not singular, and we can probe it with other probes. For example, we can use a probe composed of n D3-branes plus a single wrapped D5-brane (or, equivalently, $n+1$ wrapped D5-branes and n wrapped anti-D5-branes). It is easy to compute the metric for such a probe in the background of [15, 16], and we find

$$g_{ab} = \delta_{ab}\frac{1}{8\pi g_s}\left(2\pi n + \theta^0_B + 2Mg_s \ln\left|\frac{z}{z_0}\right|\right) \tag{2.9}$$

(we can compute this directly, or just note that this configuration may be derived from the previous one by taking $\theta_B \to \theta_B + 2\pi n$; note that the latter fact means that this is the natural probe for the theory at $|z| \simeq |z_0| e^{-n\pi/g_sM}$). Thus, for this probe the metric does not become singular until a different radial coordinate $|z| = |z_0| e^{-(2\pi n+\theta^0_B)/2Mg_s}$. This is exactly the same result that we get in the field theory for a computation of the effective metric for shifting together $n+1$ eigenvalues of the SU$(N+M)$-adjoint field φ_1 and n eigenvalues of the SU(N)-adjoint field φ_2, as expected. Similarly, for any other probes, as long as their effective metric is non singular, we find exact agreement between the 1-loop field theory result and the supergravity result.

Note that it is crucial for these agreements that the wrapped D5-brane always corresponds to the bigger gauge group (i.e. that we always have M wrapped anti-D5-branes at infinity). If we had an interpretation as in [7], in which the SU$(N+M)\times$ SU(N) group becomes an SU$(N-M)\times$ SU(N) group with the wrapped-D5-brane gauge group becoming SU$(N-M)$, the sign of the derivative of the effective metric on a wrapped-D5-brane probe would change, and there is no sign of this in the supergravity background.

The main evidence for our claim about the field theory VEVs comes from a direct computation of the VEVs of the corresponding operators in the string theory dual. The sources of the bulk fields (in the untwisted sector), such as the function Z which appears in the metric and the 5-form, are (by construction) exactly the same as the sources that would arise from a distribution of D3-branes of the form described above [16]. Thus, if we construct the theory from an SU$(N+M)\times$ SU$(N+M)$ theory as described above, we can compute the VEVs in the field theory by the usual methods of the AdS/CFT correspondence (since the background in this case is asymptotically AdS), and the VEVs will be consistent with our interpretation here, and not with an interpretation in which the VEVs vanish and there is some duality relating the theories at different scales. Note that to distinguish the two possibilities we have to use untwisted sector fields, which correspond to operators like $\mathrm{tr}(|\varphi_1|^{2n})+\mathrm{tr}(|\varphi_2|^{2n})$ (this operator is identified with some combination of the metric and 5-form fields), rather than twisted sector fields which correspond to operators like $\mathrm{tr}(|\varphi_1|^{2n})-\mathrm{tr}(|\varphi_2|^{2n})$. The latter operators do not depend on how we distribute the D3-branes so they cannot distinguish the two possibilities. Note also that even though the solution of [15, 16] is only invariant under an SU$(2)\times$SU(2) symmetry rotating the directions x_6-x_9, the source for Z is invariant under an additional U$(1)_R$ symmetry changing the phase of z. Thus, only operators which are singlets of SU$(2)\times$SU$(2)\times$U$(1)_R$ will obtain VEVs in the vacuum described by this solution. The simplest such operator is the orbifold generalization of the operator $\mathrm{tr}(2\Phi_4^2+2\Phi_5^2-\Phi_6^2-\Phi_7^2-\Phi_8^2-\Phi_9^2)$ in the $\mathcal{N}=4$ theory, where Φ_i is the scalar field whose VEV is related to the position in x_i. This operator is of the form $[2\,\mathrm{tr}(|\varphi_1|^2)+2\,\mathrm{tr}(|\varphi_2|^2)+\text{hypermultiplet fields}]$. These operators are not chiral in the $\mathcal{N}=2$ theory, but they are still present in the supergravity and their VEVs are non zero, confirming the Higgsing interpretation.

There is one problem with our interpretation, which is the main motivation for believing that a more complicated interpretation involving some strong-coupling dual might be required. The problem is that the 1-loop running of the two gauge couplings is constant in the vacuum we are considering, below the scale $|z_0|/2\pi\alpha'$

where we put the M anti-D5-branes. The SU(N) coupling becomes weaker as we go down in energy, but the SU($N+M$) coupling becomes stronger, and (using the 1-loop result) diverges at a scale quite close to $|z_0|/2\pi\alpha'$ (in fact, it diverges exactly at the scale $z_1/2\pi\alpha'$, with z_1 the position where the wrapped D5-brane kinetic terms vanish as discussed above). One possible way to interpret this is that when this coupling becomes infinite, we have to make some strong-weak coupling duality and go over to some other gauge theory, as suggested for a similar situation in [7]. However, there is no known dual for the gauge theory we are discussing, and certainly the SU($N+M$) $\times$ SU(N) and SU($N-M$) $\times$ SU(N) $\mathcal{N}=2$ theories are not equivalent in any sense (for instance, they have different dimensions for their moduli space). Our suggestion is that the gauge coupling indeed becomes large at the position of the first generalized enhançon ring, and presumably non-perturbative corrections to the beta function become important so we do not really know what happens to the running coupling below this scale. However, it seems that the running coupling as a function of energy is not directly related to anything we measure in our background; we can only directly relate measurements of $\theta(z)$ to the effective moduli space metric, as described above. Thus, it does not seem logically inconsistent to suggest that the running coupling is strong, though the effective coupling on some components of the moduli space is still (relatively) weak, as described above. In any case, even if a strong-weak coupling duality is required to understand this issue, we claim that it is not related to the reduction in the size of the gauge group, which seems to arise from a Higgs mechanism as described above.

Another apparent problem with our interpretation is that it violates the usual UV/IR matching of radial positions with energy scales in the field theory (this is also related to the previous problem). We suggest that the moduli of the theory are spread out over different radial positions, even though they are all low-energy fields in the field theory. Our justification for this is that the UV/IR correspondence is only understood in backgrounds which are asymptotically AdS, and even there the matching can be quite subtle (as in the solutions of [5],[9]) so we do not see this as a concrete problem. Note in particular that our conjecture implies that putting in a cutoff at some radial position in the string theory is not simply equivalent to an energy cutoff in the field theory, but again there are already various examples of this.

2.4 Comments on the generalized enhançon rings

Finally, let us try to discuss in more detail the physics of the generalized enhançon rings. The generalized enhançon rings occur when the kinetic term of a probe wrapped D5-brane attached to some number of D3-branes vanishes; this means that θ_B is an integer multiple of 2π. At first sight this suggests that we have tensionless

[9]In solutions of [5] involving more than one 5-brane, the massless particles of the field theory live on the 5-branes which are at finite radial positions, while the string theory fields at small radii correspond to massive particles of the field theory.

strings on the generalized enhançon ring, but we should be more careful because θ_C does not generally vanish (in the sense of being 2π times an integer) there. In fact, in the background (2.6) θ_C vanishes only on $2M$ (equally-spaced) "enhançon points" on the generalized enhançon ring, where $\theta = 2\pi(n - m\tau)$ for integer n, m. So, we only get light fields beyond the supergravity modes (and the twisted sector tensor multiplet) at these points. Note that a probe NS 5-brane (attached to some number of D3-branes) would have a vanishing kinetic term when θ_C is an integer multiple of 2π, so at these points the kinetic terms for both probes vanish. However, we do not have a direct field theory interpretation for such a probe (presumably it is related to condensing some "magnetic" degrees of freedom in the field theory). The low-energy field theory at the "enhançon points" is not clear. It seems that it is not a free field theory, so it is not clear to what extent the fields living at the enhançon point can be identified with the $2M$ "extra" moduli corresponding to breaking $\mathrm{SU}(N+M) \times \mathrm{SU}(N) \rightarrow \mathrm{SU}(N) \times \mathrm{SU}(N-M) \times G$ (where G is a subgroup of $\mathrm{U}(M) \times \mathrm{U}(M)$ of rank $2M$). By small changes in wrapped D5-brane positions we can move the "enhançon points" around, so it seems that they are related to the moduli, but the precise relation seems to be quite complicated.

The low-energy field theory living on the orbifold fixed line generically involves a free tensor multiplet, three of whose scalars correspond to blowing up the orbifold while the other two scalars are periodic and correspond to the 2-form fields integrated over the vanishing 2-cycle. For a particular value of the two periodic scalars we have additional light degrees of freedom. Note that this is not the same as the low-energy theory of the "little string theory", in which only one scalar is periodic; the "little string theory" arises in the limit $g_s \rightarrow 0$ of the $\mathbb{R}^4/\mathbb{Z}_2$ singularity, in which the periodicity of θ_B diverges (in physical units). In the background we are discussing it is clear that this periodicity is important, so one cannot discuss it in terms of the decoupled theory living on the $\mathbb{R}^4/\mathbb{Z}_2$ singularity. If we look at a particular generalized enhançon ring, θ_B is constant along it while θ_C varies as $2M$ times the angle. From the point of view of the low-energy field theory on the fixed line, one of the periodic scalar fields is linear in the angle. In this case it seems that one can discuss the physics in terms of the physics of "little string theories", but the behavior of "little string theories" in configurations of this type is not known.

3. Theories related to branes on a conifold singularity

The case of branes on a conifold was extensively discussed in [7, 8, 9, 33, 34, 35] so we will be relatively brief in reviewing it here. The low-energy theory for N D3-branes on a conifold is believed to be the low-energy limit of an $\mathrm{SU}(N) \times \mathrm{SU}(N)$ $\mathcal{N} = 1$ gauge theory with 2 pairs of bifundamental chiral multiplets, A_1, A_2 in the $(\mathbf{N}, \bar{\mathbf{N}})$ representation and B_1, B_2 in the $(\bar{\mathbf{N}}, \mathbf{N})$ representation, and with a quartic superpotential proportional to

$$W = \mathrm{tr}\left(A_1 B_1 A_2 B_2 - A_1 B_2 A_2 B_1\right). \tag{3.1}$$

This theory flows to a superconformal theory (in fact, a fixed line of superconformal theories [36]) in the IR. This SCFT is believed [33] to be dual to type-IIB string theory on the near-horizon limit of the D3-branes on the conifold, which is $AdS_5 \times T^{1,1}$. In particular, the classical moduli space of this theory is exactly N copies of the conifold (divided by S_N), and may be identified in terms of configurations with different distributions of D3-branes as described above. The classical theory has an $\mathrm{SU}(2)\times\mathrm{SU}(2)\times\mathrm{U}(1)_B\times\mathrm{U}(1)_R$ global symmetry, where the two SU(2) factors rotate the fields A_i and B_i, respectively, while under $\mathrm{U}(1)_B$ the fields A_i have positive charge and the fields B_i have negative charge. All these symmetries are unbroken also in the quantum theory (at the origin of moduli space). The $\mathrm{SU}(2)\times\mathrm{SU}(2)\times\mathrm{U}(1)_R$ symmetry is identified with the geometrical symmetry of $T^{1,1}$, while $\mathrm{U}(1)_B$ is identified with the gauge symmetry coming from the 4-form field of type-IIB string theory integrated over the 3-cycle in $T^{1,1}$.

If we add to the N D3-branes M additional D5-branes wrapped on the 2-cycle which vanishes at the conifold point, it seems that we get an $\mathrm{SU}(N+M)\times\mathrm{SU}(N)$ theory with the same field content and superpotential. As above, this theory is not asymptotically-free (although both gauge couplings are asymptotically-free at 1-loop), so it is not clear how to define it as a field theory. In [7, 9] it was shown that the near-horizon limit of this background leads to a configuration very similar to the one described in section 2, in which the 5-form flux grows logarithmically as we go to large radii, and there is a scalar field theta (coming from the integrals of the 2-forms over the 2-cycle) which also varies. Unlike the $\mathcal{N}=2$ case, in this case it is not known how to obtain this configuration from a well-defined field theory in some limit, though presumably the background is still in some sense dual to the large-N limit of this $\mathrm{SU}(N+M)\times\mathrm{SU}(N)$ theory. Another important difference from the $\mathcal{N}=2$ case is that in the solution of [7] the volume of the 2-cycle does not vanish (except at the minimal radial position), and there is no flat direction corresponding to moving around wrapped 5-branes. This corresponds to the fact that the classical moduli space of the $\mathrm{SU}(N+M)\times\mathrm{SU}(N)$ theory is still just N copies of the conifold, and there are no branches corresponding to "fractional branes".

In the quantum theory with $M>0$, the $\mathrm{U}(1)_R$ symmetry is anomalous, and the superpotential receives quantum corrections. This leads to a correction in the quantum moduli space, and it was argued in [7] that some branches of the moduli space look like D3-branes moving on the deformed conifold rather than on the conifold. This agrees with the string theory dual found in [7], which can indeed include any number of D3-branes moving on the deformed conifold (at least when their back-reaction on the background can be neglected). The deformed conifold

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no longer has the $U(1)_R$ symmetry, but it still has the $SU(2) \times SU(2)$ symmetry. When $p \equiv N \bmod M$ vanishes, the string theory dual has a good supergravity approximation, with no additional branes. For non-zero p the solution of [7] involves also (at least) p D3-branes moving in the background. These D3-branes break the $SU(2)\times SU(2)$ symmetry. In the field theory, the corresponding VEVs can be thought of as breaking the $SU(N+M) \times SU(N)$ group to $SU(N+M-p) \times SU(N-p)$, which is in the class of theories with $p = 0$. Thus, it seems to be sufficient to understand the behavior in the case of $p = 0$, and we will focus on this case from here on.

It was suggested in [7] that the interpretation of the decrease in the value of N in this case should be via Seiberg duality [37]. As in section 2, at 1-loop the gauge coupling of the $SU(N)$ factor becomes weak as we go down in energy, while that of the $SU(N+M)$ factor becomes strong and, if we define it to be constant at some UV cutoff, diverges quite close to the cutoff. One can argue [7, 8] that the exact renormalization group flow indeed causes the coupling of the $SU(N+M)$ gauge group to become very strong, and the authors of [7] argued that this is the same as the behavior of this theory at low energies, where (if we can ignore the dynamics of the $SU(N)$ gauge group) it is dual [37] to an $SU(N-M)$ gauge theory. Therefore, they argued that the reduction in N comes from a duality rather than a Higgs mechanism, and they showed that after the duality one gets a similar theory with gauge group $SU(N-M)\times SU(N)$ instead of the original $SU(N+M)\times SU(N)$, in agreement with the decrease in the 5-form flux.

The quantum-corrected moduli space of these theories seems to be very complicated, and to contain many different types of branches.[10] The $SU(N+M) \times SU(N)$ theory seems to have a branch of (complex) dimension $3N$, which looks like N D3-branes moving on the deformed conifold (and which would arise from the near-horizon limit of a configuration where the D3-branes are taken slightly off the conifold point). Describing such a branch in the dual theories with a smaller value of N is apparently quite complicated, and requires including also some massive fields in the dual theories. It was suggested in [7] that the quantum-corrected moduli space of the $SU(N+M)\times SU(N)$ theory includes M branches of dimension $3(N-kM)$ for every $k = 0, 1, 2, \ldots, [N/M]$. It was shown in [7] that the moduli space indeed had this form for $N = M$; it would be interesting to verify that this is the form of the moduli space also for $N > M$. In the particular case of $p = 0$ the smallest branches in fact have dimension one rather than zero,[11] and involve VEVs of "baryonic operators" (which we will describe below). These branches are part of the moduli space of all the $SU(N+M) \times SU(N)$ theories, for any $N = kM$, and can be described just in terms of the massless fields in all of these theories.

[10] The rest of this section is based on discussions with I. Klebanov and M. Strassler. I am grateful to I. Klebanov and M. Strassler for correcting some mistakes in an earlier version of this section.

[11] We thank M. Strassler for notifying us of this correction to the claims of [7].

To understand the interpretation of the decrease in N we need to understand which branch in the moduli space the background of [7] corresponds to. The Higgsing interpretation makes sense if the theory is in the branch with maximal dimension (the dimension is infinite in the large-N limit just like in the case of section 2), which can be directly related to some distribution of branes, as in section 2. We will call this the "mesonic branch" since the positions of D3-branes are generally related to eigenvalues of the "meson matrices" $N_{ij} \equiv A_i B_j$, which are in the adjoint of SU(N). On the other hand, the duality interpretation seems to be most useful if the theory is on the branch of lowest dimension, which can be described solely in terms of the massless fields in all the different dual theories. We will call this branch the "baryonic branch" since the computations of [7] suggest that the VEVs of meson operators vanish in this vacuum, while baryonic operators obtain VEVs.

How can we distinguish which branch are we in? We can try to compute VEVs of gauge-invariant operators in the background, but it is not clear exactly how to do this in this case (unlike section 2), since we do not know how to obtain this theory from a limit of theories which are under control. So, all we can do is try to use general arguments to constrain the VEVs. Since the background of [7] is invariant under $\mathrm{SU}(2)\times\mathrm{SU}(2)$, it is clear that only VEVs of operators invariant under this symmetry can be non zero. VEVs for eigenvalues of the mesonic operators N_{ij} break this symmetry, so it seems that we are not in a mesonic branch, but one could get around this by saying that in the large-M limit the D3-branes are smeared in an $\mathrm{SU}(2)\times\mathrm{SU}(2)$-invariant way (as in other backgrounds describing configurations on the moduli space, like the ones described in [38]). In such a distribution only $\mathrm{SU}(2)\times\mathrm{SU}(2)$-invariant operators of the form $\mathrm{tr}(\prod_l A_{i_l} B_{j_l})$ would obtain VEVs. Unfortunately, one can show that all of these operators are non chiral and do not appear in the supergravity spectrum, so it is hard to compute these VEVs (both in the field theory and in the supergravity). However, it seems that in a configuration like this, even in the large-N limit the $\mathrm{SU}(2)\times\mathrm{SU}(2)$ symmetry would be spontaneously broken, so there should be Goldstone bosons in the background, while the background of [7] seems to have a mass gap.[12] Thus, it seems more natural to identify this background with the "baryonic branch" of the field theory, which preserves the $\mathrm{SU}(2)\times\mathrm{SU}(2)$ symmetry.

In addition to the "mesonic operators" of the form $\mathrm{tr}(\prod_l N_{i_l j_l})$, the gauge theories with $p=0$ contain also "baryonic operators" of the form

$$\mathcal{B} \equiv \left[(A_i)^N\right]^{(N+M)/M}; \qquad \overline{\mathcal{B}} \equiv \left[(B_j)^N\right]^{(N+M)/M}, \tag{3.2}$$

where in $(A_i)^N$ we contract the indices to form an SU(N)-singlet in the M-th antisymmetric representation of SU($N+M$), and then we contract the SU($N+M$)

[12]Note that the backgrounds of [38] also had a mass gap in the supergravity approximation, but this approximation breaks down there and the background includes additional branes which could carry the Goldstone bosons. On the other hand, supergravity seems to be a reliable approximation in the background of [7].

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indices using an epsilon symbol (or we could do the contractions in the opposite order with the same result). All the operators of this type have a non-zero $\mathrm{U}(1)_B$ charge, and they can have various $\mathrm{SU}(2) \times \mathrm{SU}(2)$ quantum numbers depending on how we choose the SU(2) indices i. These operators include an $\mathrm{SU}(2) \times \mathrm{SU}(2)$-singlet which could acquire a VEV in the vacuum corresponding to the solution of [7], and in fact it should obtain such a VEV according to the field theory analysis of the "baryonic branch" in [7] (at least for $N = M$). Such a VEV should spontaneously break the $\mathrm{U}(1)_B$ symmetry. This $\mathrm{U}(1)_B$ symmetry is not visible in the background of [7], which is consistent with this claim.

The operators $\mathcal{B}$ may be identified (by a generalization of the analysis of [35]), in the region where the 5-form flux is $N + M$, with configurations consisting of $(N+M)/M$ D3-branes wrapped on the 3-cycle, with M strings ending on each one, and a single D5-brane wrapped on the compact 5-cycle, with the other ends of the $N+M$ strings ending on the D5-brane.[13] When one moves such a "baryon particle" in the radial direction such that the 5-form flux decreases by M and θ_B shifts by 2π, the D5-brane acquires (-1) units of D3-brane charge. Then, it can separate into a D5-brane with no D3-brane charge and an anti-D3-brane, and the anti-D3-brane can annihilate one of the D3-branes. This exactly reproduces the change in the baryon operator as one decreases N (and it is consistent with the behavior of this operator under Seiberg dualities). As we decrease the radial position to its minimal value (where the 5-form flux vanishes and the 2-cycle contracts to zero size), all the D3-branes annihilate and one is left with nothing. In the gauge theory this corresponds to the fact that the baryon is a singlet of the final $\mathrm{SU}(M)$ gauge group, and it proves that indeed the baryon does not carry any conserved charge in the background of [7], so the $\mathrm{U}(1)_B$ is indeed broken. Since the $\mathrm{U}(1)_B$ is spontaneously broken in the field theory, there should be a corresponding Goldstone boson in the background, and it would be interesting to identify this state. Note that unlike the $\mathrm{SU}(2) \times \mathrm{SU}(2)$ Goldstone bosons, this state should couple only to non-perturbative states like the one described above, so it is more complicated to identify it, and it would not lead to massless poles in the scattering of states corresponding to supergravity fields.

Thus, it seems that the background of [7] corresponds to the "baryonic branch" of the field theory.[14] Therefore, the decrease in N in this background seems to correspond to a duality interpretation rather than to a Higgsing interpretation.[15]

[13]The number of strings ending on each brane is determined by charge conservation after taking into account the RR fields in the background.

[14]Note that Seiberg duality applies also to baryonic branches of the moduli space when all quantum corrections are properly taken into account [39].

[15]A relation between Seiberg duality and Higgsing in a "baryonic branch" appeared in the work of [40] on mass-deformed $\mathcal{N} = 2$ gauge theories. It would be interesting to understand if there is any relation between those results and our results here; there could perhaps be a direct connection since naively when we add mass terms to the $\mathcal{N} = 2$ theory described in section 2 we get the theory described in this section.

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Note that this identification means that configurations with different numbers of D3-branes moving in the background of [7] are generally not on the same branch of the moduli space.

If our identification is correct, our discussion suggests that there should also be generalizations of the background of [7] which would correspond to other branches of the moduli space, including branches where a Higgs interpretation would be more appropriate. Another solution with the same asymptotic behavior was found in [18]. This solution involves a resolved conifold rather than the deformed conifold which appears in [7]. The solution of [18] is singular and has a repulson in it, but it might be possible to resolve this singularity by the enhançon mechanism and replace it with some distribution of branes which is a consistent background of string theory. The solution of [18] was claimed in [14] not to be supersymmetric, based on the behavior of the 3-form fields in that solution. However, this could change once the singularity is resolved, since we expect the enhançon to be a source for the 3-form fields and to change the metric (presumably the enhançon consists of D5-branes wrapped on the 2-cycle). So, we will assume here that there is a resolution of the singularity (or a generalization of the solution of [18]) which is supersymmetric. In such a case this background should be a different vacuum of the field theory we discussed in this section. We conjecture that this background may correspond to a point on the "mesonic branch" of the moduli space, where there is a distribution of the eigenvalues of N_{ij} which matches the source for the 5-form field in the solution. Again, we do not know how to verify this conjecture directly, but it seems likely that this should be the case, since already in the conformal case [22] resolving the conifold was interpreted as giving VEVs to the mesonic operators. Note that, assuming that the resolution of the singularity in this background involves an enhançon, it may not have a mass gap, which would be consistent with our expectation for having $\mathrm{SU}(2) \times \mathrm{SU}(2)$ Goldstone bosons in the background corresponding to the "mesonic branch". As additional evidence we note that in the resolved conifold the 2-cycle never collapses, so we may not be able to annihilate to nothing the baryon particle described above, suggesting that perhaps in this background the $\mathrm{U}(1)_B$ symmetry is unbroken. All this is very weak evidence for our conjecture; it would be interesting to perform more tests of this identification.

We should emphasize that whenever we are talking of an interpretation for the decrease in N, we are really talking about what happens in the weakly coupled theory, since only there we can really see that we have some particular gauge group, and what happens to it when we change the energy scale. The supergravity backgrounds all correspond to strong coupling (large $g_s M$), so the more precise statement of our suggestion is that if we continue these backgrounds to weak coupling, then the solution of [7] would have an interpretation via Seiberg duality, while that of [18] might have an interpretation in terms of a Higgs mechanism. It is not clear what these interpretations mean directly in the strongly coupled theory. Naively the change in the theta angle can be interpreted as a running coupling, which was interpreted in [7] as supporting the duality interpretation, but we saw in section 2 that such an interpretation does not always seem to work.

It would be very interesting to understand better the exact quantum superpotential in these theories, in order to verify the claims of [7] about the quantum moduli space. We gave evidence for identifying particular branches in this moduli space with particular string theory solutions, but our evidence is far from conclusive, and it would be nice to substantiate these identifications. In particular, it would be interesting to close the possible loopholes in our identification of the configuration of [7] with the "baryonic branch" in the moduli space, for example by identifying properly the $\mathrm{U}(1)_B$ Goldstone boson (which presumably comes from the RR fields in the background of [7]), and to test our suggestion that the configuration of [18] could correspond to the "mesonic branch" of the same theory. Of course, this requires a resolution of the singularity in the solution of [18]. Perhaps there are different ways of resolving this singularity, that would correspond to different branches in the field theory moduli space, or there could be additional solutions (perhaps involving generalizations of the deformed conifold and resolved conifold metrics) that would correspond to the other branches. Alternatively, it is possible that there is no simple string theory dual for the "mesonic branches" of the moduli space.

Hopefully, the results we presented here will enable a better understanding of holography in backgrounds with varying flux. In particular, it would be nice to have a better understanding of exactly how to define the corresponding large-N field theories directly in field theory terms. We saw that such an understanding exists in the $\mathcal{N} = 2$ case, but it does not yet exist in the $\mathcal{N} = 1$ case as far as we know.

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